The Excited Spin State of Comet 2P/Encke

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Abstract

Ways to rationalize the different periods (*e.g.,* 15.08 hr, Luu and Jewitt 1990; 11.01 hr, Fernández *et al.* 2004, Lowry *et al.* 2003) seen in near aphelion R-band light curves of comet 2P/Encke are explored. We show that the comet is usually active at aphelion and it's observed light curves contain signal from both the nucleus and an unresolved coma. The coma contribution to the observed brightness is generally found to dominate with the nucleus providing from 28 to 87% of the total brightness. The amplitude of the observed *variations* cannot be explained by the nucleus alone and are due to coma activity. We show that some seven periodicities exist in the observed light curves at various times and that this is likely the result of an active nucleus spinning in an excited spin state. The changing periodicities are probably due to changes in the relative strengths of the active areas. We work out possible excited states based on experience with model light curves and by using an analogy to light curve observations of comet 1P/Halley for which the spin state has been separately determined from spacecraft observations. There is a possibility of a fully relaxed principal axis spin state (0.538 d^{-1}; P = 44.6hrs) but, because it provides a poorer fit to the observed periodicities than the best fit excited state together with the absence of a peak near 1.08 d^{-1} ($2f_{\varphi}$) in the frequency spectrum of the Fernandez *et al.* (2000) thermal IR lightcurve, we consider it unlikely. Both SAM and LAM excited states are allowed by the underlying periodicities and additional information is needed to choose between these. Our choice of a low excitation SAM state, *i.e.,* one in which the instantaneous spin axis nutates around the total angular momentum vector in a motion that is characterized by limited angular oscillations around the long axis, is based on Sekanina's (1988*a, b*) interpretation of the fan coma that this comet often displays. We argue that possible LAM states are excluded either because they are too difficult to excite or because they would be inconsistent with the formation of the observed fan morphology.

Two possible SAM states emerge that provide good fits to the observed periodicities, one with a precessional frequency for the long axis about the total angular momentum vector of 1.614 d^{-1} ($P\phi$ = 14.9 h) and an oscillation frequency around the long axis of 0.539 d^{-1} ($P\psi$ = 44.5 h) and

a second with a precessional frequency of 2.162 d^{-1} ($P\phi$ = 11.1 h) combined with an oscillation around the long axis of 0.502 d^{-1} ($P\psi$ = 47.8 h). While either solution is possible, the latter is, in a least squares sense, more likely to be the actual spin state. In both cases the direction of the total angular momentum vector (α_M, δ_M *[J2000] = 198.6, -0.3 deg)* is assumed to be defined by the evolving geometry and morphology of the coma (Sekanina 1988*a, b*; Festou and Barale 2000).

We discuss the possible locations of the primary active areas found by Sekanina (*1988a, b*) and, while they are at high cometographic latitudes, they do not have to be physically located close the region were the axis of maximum moment of inertia pierces the surface (*i.e.*, at high cometocentric latitude). We offer a new interpretation of the 10.7 μm data by Fernández *et al*. (2000) which yields an axial ratio a/b = 2.04. This, with the two SAM states that we have found, requires that b/c > 1.18 or > 1.09 implying a significant asymmetry in the shape of the elongated nucleus. For the observed fan morphology to be maintained, the true axial ratio b/c cannot be much larger than these limiting values otherwise the amplitude of the oscillation about the long axis becomes too large and the fan morphology would be destroyed. The precise phasing of the spin modes, *i.e.* the value of the Euler angles at a particular time, is not determinable from the current data set, but a set of well sampled thermal infrared observations of the nucleus covering many periods and a wide range of observing geometries could provide this information in the future as well as clearly distinguishing between the two excited spin states.

I. INTRODUCTION

In a companion paper, Fernández *et al.* (2004) compare the periodicities found in near-aphelion light curves of comet 2P/Encke in 2000 and 2001 with the results of the earlier (in the 1980's) R-band investigations of Jewitt and Meech (1987), Luu and Jewitt (1990), and also the work at 10.7 μm by Fernandez *et al.* (2000). Although not visually evident in the image data, a substantial amount of light from unresolved coma was detected in the 2000 and 2001 near-aphelion data and two periodicities were found at $f_0 = 2.16\ d^{-1}$ (11.1 hrs) and $f_1 = 4.35\ d^{-1}$ (5.52 hrs). The 1980's data, which consistently show a strong periodicity near 3.2 d^{-1}, have been confidently interpreted to indicate a 15.08 h (1.59 d^{-1}) spin period (*e.g.* see the reviews by Belton, 1991, and Jewitt, 1997) for the rotation of the comet's nucleus, in apparent conflict with f_0 and f_1. In addition, Lowry *et al.* (2003) have independently analyzed their data of September 2002, to find a period of 11.01 hrs (2.18 d^{-1}) and to make a visual detection of the presence of a dust trail.

In this work we investigate how the various periodicities that appear in these data sets might be rationalized in terms of models of the spin of the nucleus. One that we consider is a fully relaxed rotator spinning around the principal axis of largest moment of inertia, while other models visualize an assumed elongate nucleus in an excited state of free-precession. Such a spin state could be induced in a nucleus that has a roughly axisymmetric shape and that has active areas which cause reaction torques strong enough to affect the spin in times that are long compared to the instantaneous spin period thus leading to relatively stable excited motion over orbital timescales. This latter kind of model has been successfully used in the past to model the spin and shape of comet 1P/Halley's nucleus (Belton *et al.*, 1991; Samarasinha and A'Hearn, 1991). By way of explanation, excited spin states of a freely precessing rigid body (*cf.* Belton 1991, or Samarasinha and A'Hearn 1991) are divided into two types: a low energy short axis mode (SAM) and a higher energy long axis mode (LAM). In both cases the instantaneous spin vector freely precesses around the total angular momentum vector. In the lower energy states, the motion is characterized by an oscillatory motion around the long axis (SAM), while in the higher energy states there are complete rotations around the long axis (LAM).

We first consider the origin of the light scattered from the comet when it is near-aphelion. We find that there must be a variable and spatially unresolved coma contribution that usually dominates the signature of the nucleus, *i.e.,* the observed periodic modulation in the comet's brightness is dominated by cometary activity rather than light scattered directly from the surface. We then examine the periodicities found in the various data sets and present a reinterpretation of the older data of Jewitt and Meech (1987), Luu and Jewitt (1990), and Fernández *et al.* (2000). We show that the periodicities latent in all of these data appear to be roughly harmonically related. Three possible spin models are isolated, one a fully relaxed system in principal axis rotation and two in excited states. After a short review of what is known about the direction of the spin axis based on the distinctive fan shaped coma morphology that was analyzed by Sekanina (1987, 1988*a, b,* 1991*a*) and Festou and Barale (2000), we show that the periodicities in 2P/Encke are most reasonably interpreted in terms of a low energy, short axis mode (SAM) spin state.

II. ORIGIN OF THE LIGHT VARIATIONS NEAR APHELION

For this study it is important to determine the origin of the brightness variations observed in 2P/Encke when it is near aphelion, *i.e.,* whether the observed periodicities are characteristic of activity on the nucleus or a reflection of its shape. 2P/Encke's current aphelion and perihelion distances are 4.09 and 0.34 AU respectively. Early photographic studies of distant comets (*i.e*., comets at heliocentric distances greater than 3 AU) by Roemer (1962) showed that comets are often active at large distances from the sun. In her observations this activity was obvious in the form of spatially resolved comae and tails. Later Barker *et al*. (1981) provided evidence of substantial outbursts in the brightness's of two Jupiter family comets, 10P/Tempel 2 and 2P/Encke, while they were far from the Sun (3.2 and 3.9 AU respectively). In this case there was no obvious indication of coma in the data showing that it is possible for distant comets to be active even though their images remain stellar-like.

In their interpretation of CCD observations of 2P/Encke at 3.1 and 4.1 AU, Jewitt and Meech (1987), and later Luu and Jewitt (1990), argued that the brightness variations are the signature of the spinning nucleus, with, at most, minor contamination from coma. Their arguments include the following points: "Transient nucleus activity would be aperiodic"; "The surface–brightness profile conveys no hint of coma;" "The *persistence* (our italics) of the same periodicities in the photometry of P/Encke from 1985, 1986, and 1988 strongly suggests that the periodicity is caused by the rotation of the nucleus;" and "This simple, monotonic photometric behavior [of the observed photometric phase curve] strongly argues against the possibility that the cross section of P/Encke is dominated by coma." These arguments were later questioned by Sekanina (1991*b*) who forcefully pointed out problems of consistency with the run of brightness amplitudes when the observations are interpreted as the signature of scattering from the nucleus.

Today the arguments of Jewitt and Meech (1987) and Luu and Jewitt (1990) no longer appear to carry the force that they once did. Continuous activity at large heliocentric distances is now a well observed phenomenon and, indeed, Meech *et al.* (2001) have noted from their large database that 2P/Encke "…is definitely active near every aphelion" ; observations of outbursts without changing the stellar-like appearance of a distant comet exist (Barker *et al*., 1981) and, as a result of the work of Fernández *et al.* (2000), our appreciation of the phase curve of Encke has radically changed since the initial study of Luu and Jewitt (1990) allowing for substantial coma contribution in the latter's observations. Finally, as outlined in the following sections, and shown by Lowry *et al.* (2003) and Fernández *et al.* (2004), the *persistence* of the periodicities seen in the 1980's has now ended with the emergence of new periodicities.

Based on work by Delsemme (1982) it was believed that there was minimal if any water production at this heliocentric distance and beyond, and except at times of an unusual outburst, periodic comets show little or no evidence of coma and cometary activity. However, as summarized by Meech & Svoren (2004), water ice sublimation can lift optically significant grains off the surface of a nucleus as far out as 5-6 AU from the sun, and other processes (such as the annealing of amorphous water ice) can be responsible for significant activity out to even larger distances. However, the observational record is sparse and only a few periodic comets have been studied in detail while near aphelion. Several of these show no indication of activity when near aphelion. 9P/Tempel 1, for which an extensive data set beyond 4 AU now exists (Meech *et al.*, 2002), is an example of such a case as is 10P/Tempel 2 (Mueller and Ferrin, 1996). In the database of 10 periodic comets observed by Meech (unpublished) near aphelion, eight showed no evidence of activity when near aphelion.

Nevertheless, some periodic comets are clearly active at this distance, *e.g.*, 1P/Halley's pre-perihelion brightness was rising fast at this distance (Wyckoff *et al.* 1985) and was well elevated over nucleus values; also, 2P/Encke was observed by Barker *et al.* (1981) to be fading from a outburst at 3.9 AU, dimming by 1.1 magnitudes during the observations while the appearance of the comet remained stellar like. Meech (unpublished) has seen activity in 65P/Gunn and 74P/Smirnova-Chernykh when near aphelion.

Table 1 summarizes the observational data and orbital parameters for 2P/Encke that are relevant to this study and in Figure 1 we show the orbital geometry appropriate to the observations. Figure 1 shows the Earth directions corresponding to different datasets considered in this analysis. Also shown are the sun direction at aphelion and the direction for the rotational angular momentum vector. While the Earth directions and the sun direction at aphelion can be accurately determined, we believe the uncertainty for the rotational angular momentum vector is at least 10 degrees. This figure helps one to visualize how the projected nuclear cross section (both the mean value as well as the amplitude) may have changed in different datasets.

The R-band observations for study of the rotation were taken in a range of heliocentric distance between 3.13 and 4.06 AU and if only light from the nucleus were being observed, the values of the mean absolute magnitude should vary little unless major changes in the rotational angular momentum direction occur from aphelion to aphelion. We assume that such radical changes in spin geometry do not occur on such short timescales at present, although Sekanina (1991*a*) has shown that much slower changes may indeed have occurred at 1-3 deg/revolution in the past. In reality the magnitudes, which are good to ± 0.02 mag or better, fluctuate between 15.1 and 13.1 with no clear trend with heliocentric distance. This suggests that the signature of the nucleus is embedded in variable levels of light from an unresolved coma. Figure 2 shows a heliocentric light curve for 2P/Encke observed by Meech *et al.* (2001) that clearly shows the variability of the comet near aphelion.

Estimates of the relative contributions of coma and nucleus can be obtained with the help of the 10.7 μm work of Fernández *et al.* (2000). They find, after using a coma modeling technique to make corrections for a small amount of coma (estimated to be at about the 12% level) and using the Standard Thermal Model for thermal emission (appropriate for the low thermal inertias [Julian *et al.* 2002] of cometary material), an effective mean radius for the nucleus of 2.4 $\pm$ 0.3 km. The R-band geometric albedo of the nucleus probably lies in the range of the measured values of other cometary nuclei, *i.e.,* between 0.02 and 0.05 (Jewitt, 1997; Lamy *et al.,* 2004). This taken together with Fernández *et al's* radius implies a mean absolute R magnitude of 2P/Encke's nucleus in the range 14.56 < $H_R(1,1,0)$ < 16.72 where we have also included a 0.31 mag uncertainty that arises from the orientation of the spin vector when interpreting Fernandez et al's effective mean magnitude in terms of an elongate object. Following Fernandez *et al.* (2000) the probable value of $H_R(1,1,0)$ is 15.2 mag. We note that a single R band HST datum, taken when the comet was at a heliocentric distance of 0.942 AU (Fernández *et al.,* 2000), and when corrected for coma contributions and a phase law of 0.06 mag/deg (the phase angle for this observation was large at 106.2 deg and the rotational phase was undetermined), leads to an estimate for the absolute magnitude of the nucleus near 15.0. Fig. 2 shows that these estimates of the mean nucleus magnitude are in concert with behavior of the heliocentric light curve.

With the exceptions in September and October of 2001 (Table 1), the observed mean absolute magnitude is consistently brighter than the top of the above brightness range (*i.e.,* H_R = 14.56) in all of the data. This judgement is somewhat dependent on the uncertainties in the phase law at small phase angles. Fernandez *et al.* (2000) do not assess the uncertainty in β (= 0.06 mag.deg), the linear phase law coefficient, in their determination of this quantity for this comet, but an estimate of $\pm$ 0.02 mag/deg is sometimes used (*e.g.,* Licandro etal., 2000) as typical for cometary nuclei. For the range of phase angles in the 2P/Encke R-band observations (0.9 – 15.4 deg) this would imply absolute magnitude uncertainties from $\pm$ 0.02 to $\pm$ 0.31 mag. While these uncertainties could either increase or decrease the contributions of the nucleus to total light shown in Table 1 the effect, barring the exceptions noted above, should be small. Thus we conclude that coma contamination is likely present in most, if not all, of the data sets. The question to be faced is whether this contamination is so pervasive that the variations in brightness are primarily due to cometary activity or the changing geometry of the spinning nucleus.

In the seventh column of Table 1 we give the range of nucleus to total brightness for the range of $H_R(1,1,0)$ noted above. This is calculated as follows: if I_N is the contribution of light from the nucleus and I_C is the light from the unresolved coma, then providing optical depth issues can be ignored and from the definition of magnitudes, the ratio of I_N to total light can be calculated from

$$H_N - H_T = 2.5 \log [(I_N + I_C)/I_N] \qquad (1)$$

where H_N is the assumed absolute magnitude of the nucleus and H_T is the observed absolute magnitude. In parentheses in column 7 we also give the same quantity assuming that the mean nucleus absolute magnitude has the probable value of 15.2. Our interpretation of these figures is that the nucleus contributes only a fraction to the total light, usually more than 28% but rarely over 87% at these heliocentric distances.

We now enquire what modulation range of the total brightness could be caused by the rotation of the nucleus while assuming there is no modulation in the coma contribution. If the nucleus modulation is less than what is actually observed then we have a strong case that the coma itself is being modulated by the underlying rotation. The thermal infrared observations of Fernández *et al.* (2000) show an amplitude (peak to valley) of about 0.7 magnitudes in the nucleus emission suggesting an elongated nucleus. In order to estimate the contribution of the nucleus brightness variations in the near aphelion observations we also require knowledge of the spin pole direction and the spin state so that the aspect angle for the various observational geometries can be calculated. Since the details of 2P/Encke's spin state are not *a priori* available we make estimates based on an assumed principal axis rotator in which the spin axis and angular momentum vector are parallel. Because our preferred spin states turn out to be low-energy SAM states this assumption should not profoundly affect the conclusions found below. Values for the spin axis direction have been estimated by Festou and Barale (2000) and Sekanina (1987, 1988*a, b,* 1991*a*) and although there is, in our opinion, considerable uncertainty in the results, substantial confirmation of their work comes from the fact that they find similar pole directions and that these directions are consistent with observational constraints set by the infrared observations of Reach *et al.* (2000). Using two different approaches these two studies yield poles at (RA (1950) = 198 deg; Dec = 0 deg) and (RA (1950) = 205 deg; Dec = 2 deg). With Barale and Festou's pole (see Sec.V below), the aspect angle, *i.e.,* the angle between the line of sight and the pole direction, for the Fernández *et al.* (2000) 10.7 μm observations is 66.0 deg. This implies a full amplitude (*i.e.,* as seen with an aspect angle of 90 deg) of ~0.77 mag, or an axial ratio of a/b ~ 2.04 assuming a near-prolate ellipsoidal shape (*i.e.,* a >b ~ c where a, b, c are the lengths of the principal axes. NB. This simple geometric correction for aspect angle follows Fernandez *et al.* (2000) (their Eq. 3 with b=c) and is itself an approximation. It accounts for the assumed ellipsoidal shape of the body but ignores the effects of limb darkening and other illumination factors). We can calculate the aspect angle for each of the R-band observations and thereby estimate what the amplitude of the nucleus variations should be. These are shown in the two right hand columns of Table 1. In each case the prediction for the diluted nucleus amplitude is considerably less than the actual observed light curve amplitude, but nevertheless non-negligible. *Our interpretation is that the light variations seen in the near-aphelion observations are often dominated by a variable coma responding to the underlying nucleus rotation, but with non-negligible contributions being made by the nucleus itself.* We note that the phasing of the light variations due to the coma is not necessarily the same as the light scattered from the nucleus and, because of the possibility of multiple active areas, their respective dominant periodicities, while harmonically related, need not be the same. In addition, as the spatial distribution of active areas evolve and/or their relative contributions to the coma change, the harmonic content of the light curve can be expected to change. *We propose that this is the origin of the change in dominant periodicities seen in observations of 2P/Encke.*

III. PERIODICITIES IN THE LIGHT CURVES OF COMET 2P/ENCKE

In Fig. 3 we show the results of a Fourier analysis of the light curve data of Jewitt and Meech (1987), Luu and Jewitt (1990) and Fernández *et al*. (2000). The Jewitt and Meech 2.1m and 4m telescope observations have been separated in this presentation. The WindowCLEAN algorithm was used to do the analysis because in most cases it removes aliases due to the particular way the data was sampled and which would have been present in an uncleaned Fourier transform. It is also an appropriate tool for non-sinusoidal (but periodic) light curves. Our application of this technique is the same as that described in the accompanying paper by Fernández *et al.* (2004). For direct comparison, we reproduce in Fig. 4 the results of the analysis by Fernández *et al.* (2004) on data taken in Aug. 2001 and Sept. 2002 showing different periodicities.

Figs. 3a shows the apparent persistence of the frequency near 3.2 d^{-1} in the four data sets taken between 1986 and 1997 while Figs. 4a and 4b show the spectral emphasis changing to frequencies near 2.2 d^{-1} and 4.3 d^{-1} in more recent data. The strongest frequencies (between 0 and 10 d^{-1}) in each spectrum are collected in Table 2. Visual inspection of the light curve data indicates the probable absence of frequencies higher than 10 d^{-1}. In line 9 of this table we give average estimates of the full set of frequencies that appear with any strength during the entire period from 1986 through 2002. To obtain these frequencies for the data between 1985 and 1997 we find the frequency that gives the best least squares fit to the data using the WindowCLEAN spectra as a guide. The data are then whitened of this frequency, *i.e.,* a least squares fit sinusoidal variation to the data at this frequency is subtracted, and the procedure repeated to expose any underlying frequencies (Fig. 3b). Occasionally peaks occur at a value of x.00 d^{-1} where x is an integer, *e.g.*, the peak at 4.0 d^{-1} in the analysis of the Oct/Nov 1986 data of Jewitt and Meech (1987) seen in Fig. 3. These peaks have been discarded as being a spurious result of the transform of daily sampled data. It should be pointed out that even for a principal-axis rotator, undersampling of data could produce a pair of signatures such as 2.2 d^{-1} and 3.2 d^{-1} to appear in the light curve (D. Schleicher, private communication). Careful examinations show this is true for

Jewitt and Meech (1987) data as they themselves admit. However, all datasets which show these frequency signatures do not suffer from this undersampling effect. Therefore, we conclude that the consistent presence of these frequencies in multiple light curves (*cf.* Table 2) cannot be attributed to an undersampling of the data.

For the Fernández *et al.* (2004) data we have taken their results as reported. The full half-width of the peaks in the spectra range average 0.39 d^{-1} and, following the argument of Mueller and Ferrin (1996), this implies a standard deviation of around $\pm$ 0.16 d^{-1}. We believe that this represents a conservative estimate of the errors that are implicit in most of the listed frequencies. Where frequencies are particularly poorly determined we have marked them with a colon and given them half weight. In line 9 of Table 2 the listed uncertainty of the average frequencies is represented by the uncertainty of the mean.

As a guide to our interpretation of this data we consider the case of the periodicities found in the many light curves of 1P/Halley (Belton, 1990) and also the results of experience with extracting frequencies from model light curves (Kaasalainen 2001; Mueller *et al.* 2000). In what follows we consistently use the nomenclature of Samarasinha and A'Hearn (1991). This reference defines the Euler angles (ϕ, ψ, θ) that describe spin states (*cf.* their Fig. A1. Note that these angles are defined for the cometary problem and are not the same set that is commonly used in physics texts). In Table 3 we display the results of an analysis of five light curves of 1P/Halley. This material was taken from Table II in Belton (1990) and transcribed into the same format as Table 2. In the case of 1P/Halley we are reasonably sure that the spin of 1P/Halley is excited and that the nucleus is spinning in a LAM state (Belton *et al*. 1991; Samarasinha and A'Hearn 1991) as a result of an analysis of the Giotto and Vega images of the nucleus (*cf.* Keller *et al.* (1994) and Szegö *et al.* (1994) for an excellent overview of the observations). We note that Szegö *et al.* prefer a different solution to ours; however their solution probably cannot, and has not, to our knowledge, been shown to satisfy any of the available ground-based data (Samarasinha *et al.* 2004). The LAM state is characterized by two frequencies f_ϕ = 0.27 d^{-1} (P = 3.69 d) and f_ψ = 0.14 d^{-1} (P_ψ =7.1 d). In line 8 of Table 3 we assign the different frequency modes that appear in the light curves using these basic frequencies. A least squares fit to the ground-based periodicities then leads to slightly different values for these periodicities. It is interesting to speculate that these ground-based values are more accurate than those inferred from the spacecraft images. They do, however, provide confirmation for the spin state that was found by Belton *et al.* (1991). In this interpretation, the famous 7.4 d periodicity found by Millis and Schleicher (1986) turns out to be the (f_ϕ - f_ψ) mode. This table illustrates the dangers of relying on the use of a single source of data when deriving a spin model for a comet: In the last two lines we show a list of plausible assignments to the observed frequencies in terms of a simple principal axis rotation model with $f_\phi = 0.142\ d^{-1}$ (P = 7.05 d) as the basic rotation frequency that minimizes, in a least squares sense, the deviations from the set of collected frequencies. Given the agreement with the observed frequencies it is almost certain that, without the benefit of the images from the Giotto and Vega missions, this (incorrect) interpretation would have become the accepted rotation state. It is also probably the case that had the 7.4 day periodicity not been found that the solutions of Belton *et al*. (1991) and Samarasinha and A'Hearn (1991) would not have been devised and another solution based on a precession period of 2.2 days (*cf.* Szegö *et al.,* 1994, for efforts to incorporate a 2.2 day periodicity) would have become accepted (we thank D. Schleicher for pointing this out to us). The presence of very high harmonics is interpreted in both cases as the result of multiple active areas on the nucleus but their precise origin remains puzzling.

In addition to the 1P/Halley example above, a number of investigations have been reported of the periodicities that appear in light curves of model elongated comet nuclei spinning in excited states (Belton 1991; Mueller *et al.* 2002; Kaasalainen 2001). If the comet is active then the harmonic mix that is seen somehow reflects the distribution and relative strengths of active areas on the nucleus. If the nucleus is spinning in a fully relaxed state then localized activity will induce harmonics (including, but not necessarily, the fundamental) of a single frequency in the light curve. If the nucleus is in an excited state, spectral signatures that compound two independent frequencies will be seen. Thus modes such as f_ϕ, f_ψ, f_ϕ-f_ψ, f_ϕ+f_ψ, … nf_ϕ-mf_ψ, nf_ϕ+mf_ψ … can be expected to appear (n and m can be either positive or negative integers including zero). For example, in a study of the nucleus of comet 29P/Schwassmann-Wachmann 1, Meech *et al*.

(1993) found evidence of three periodicities in the light curve that they associated with spin modes: $2f_\phi$, $2(f_\phi + f_\psi)$, and $2(2f_\phi + f_\psi)$. Generally knowledge of these basic periodicities is not sufficient to distinguish between LAM and SAM states. Only if $f_\phi / f_\psi < 1$ (Samarasinha and A'Hearn, 1991) can a LAM state be assured (SAM states require $f_\phi / f_\psi > 1$ but cannot be distinguished from LAM states that also exist for this condition).

It is unlikely that small scale albedo variations on the surface of a nucleus have an appreciable effect on the unresolved nucleus light curve. While comet 19P/Borrelly, which was visited by the Deep-Space 1 spacecraft, is reported to show evidence of albedo spots on its surface (albedo ranging between 0.01 and 0.035 in the visible [Soderblom *et al.* 2004]), the light curve has not been well studied. There are, however, a handful of comets for which both visible and thermal IR light curves exist, and these do not show out of phase variations which would be indicative of albedo effects in the light curve. Thus, if a comet is not active and the light curve features only the signature of light scattered directly off the nucleus, our *experience* with the few comets that have both visible and thermal IR light curves, shows that albedo contrasts across the surface have little influence on the visual light curve and the observed brightness modulation primarily reflects the shape of the nucleus. If elongate, a fully relaxed rotator will feature a single frequency (possibly with weak harmonics) at twice the rotational frequency. If the spin is excited such a rotator will feature frequencies that are a compounded of two independent frequencies. Thus Mueller *et al.* (2002) and Kaasalainen (2001) find that $2f_\phi$ and $2(f_\phi + f_\psi)$ are usually present for LAM and SAM states but other modes are possible. A clear detection of two independent frequencies would seem to guarantee easy identification of excited rotators. However, Samarasinha and Belton (1995) have found that in the evolution of excited spin states of elongated objects there is a tendency for them to spend much time in states where the two frequencies are roughly commensurate. 1P/Halley is apparently a case like this and, as we have seen in Table 3, it is not easy to distinguish between fully relaxed and excited rotation just on the basis of light curve periodicities alone.

IV. PERIODICITIES IN SPIN MODELS FOR 2P/ENCKE

Our discussion of spin models for 2P/Encke starts with recognition that only in the 10.7 μm light curve data of Fernández *et al.* (2000) is a low level of coma contamination guaranteed. In this data the shape of the nucleus should dominate the modulation and the second harmonic of the precessional frequency, *i.e.,* $2f_\varphi$, is expected to predominant in the spectrum. Reference to Fig. 3 (or Table 2) shows that this implies that $2f_\varphi \approx 4.27\ d^{-1}$ since this peak is the strongest in the transformed data. This was not the conclusion reached by Fernández *et al.* (2000) who relied on a string-length analysis to locate periodicities in the data and a presentation that emphasized the lower frequencies in the data. Thus they chose the spectral peak with less power near $3.27\ d^{-1}$ as $2f_\varphi$ (*cf.* Fig. 3). This seemed very reasonable at the time because of the earlier published results of Luu and Jewitt (1990) that supported this conclusion.

According to our experience with modeling light curves discussed above, if the nucleus is in an excited state, we might also expect to see a spectral peak in the 10.7 μm data that would correspond to the $2(f_\phi + f_\psi)$ mode. In Fig. 3b there is a spectral peak appears near $5.25\ d^{-1}$ that could correspond to this mode if the peak seen in several data sets near $0.5\ d^{-1}$ is identified with f_ψ. Following this logic the peak near $3.2\ d^{-1}$ can then be identified as the $2(f_\phi - f_\psi)$ mode.

This *prima facie* case for assigning modes to an excited spin state for 2P/Encke is not necessarily unique and the reliance on the relative power in the two strongest peaks to choose the basic precessional periodicity could be misleading (Foster, 1995). The significance of the wider range of periodicies that occur in the complete data set (*i.e.,* line 9 of Table 2) must also be explored. To do this we have considered a range of models for the periodicities, several equivalent to excited states and one corresponding to principal axis rotation. We solve for the basic periodicities f_ϕ and f_ψ by a least squares procedure. In the excited case, the i^{th} model frequency is given by $f_i = N_i*f_\phi + M_i*f_\psi$ where N_i and M_i are integers (either positive or negative or zero) that define the model. In the fully relaxed case $f_i = N_i*f_\phi$. The objective is to find a set of N_i and M_i that best fit the full set of average frequencies in line 9 of Table 2. To evaluate the goodness of fit of a model we have used, in the spirit of least squares, two simple statistics that weight the observed frequencies somewhat differently:

$$G = (1/n)\Sigma(f_i - fobs_i)^2 \tag{3}$$

$$H = (1/n)\Sigma(1 - fobs_i/f_i)^2 \tag{4}$$

Where n is the number of observed frequencies (in this case n = 7), $fobs_i$ are the observed frequencies, and the summation is taken over all *i* observations. The more probable models will have lower values of G and H. The reason for using two differently weighted statistics (H is weighted to the lower frequencies) is simply to reinforce the choice between the models.

In Table 4 we show a few results from this model fitting process that gives the smallest values of the G and H statistics. The same models are selected by both statistics suggesting that the method is not overly sensitive to the choice of the functional form of the statistic. Model B has the lowest values of G and H and this selects between the very similar models B and C. However, the rather different model A has values of G and H almost the same as B and C. All other models that were tried had larger values and so represent poorer fits. Perhaps the first observation that should be made about the results in this table is that the best excited rotation model (Excited B) provides a better fit to the data than the fully relaxed rotator by a considerable factor.While this is perhaps not too surprising (there are more free parameters in the excited case), the margin of a factor of 5.8 in G and 2.4 in H is quite large. Primarily for this reason, but also because of the lack and evidence of a strong response in the frequency spectrum of the 1997 thermal IR observations (see fig 3a and 3b) of Fernandez et al. (2000) at 1.08 d^{-1} ($2f_\varphi$), we find that a fully relaxed principal rotation state is not likely for this comet.

To decide between the excited models A and B, given that both fit the data on periodicities essentially as well, is exceedingly difficult with a decision essentially hanging on an assessment of the relative power seen in the spectral peaks near 4.27 and 3.27 d^{-1} in the WindowCLEAN analysis of the Fernández *et al*. (2000) data (bottom panel, Fig. 3a). The distribution of power in a Fourier transform is a complex issue and may depend on the sampling of the data as well as the intrinsic strengths of the component frequencies. Given that the Fernández *et al*. (2000) 10.7μm data set is small and sparsely sampled over a relative short period of time, the relative power in the clean spectrum in Fig. 3a may not be a reliable guide. *We* *therefore take both excited models A and B with f_ϕ = 1.614 d^{-1} (Pϕ = 14.9 h) and f_ψ = 0.539 d^{-1} (Pψ = 44.5 h) and f_ϕ = 2.162 d^{-1} (Pϕ = 11.1 h) and f_ψ = 0.502 d^{-1} (Pψ = 47.8 h) respectively as probable representations of the excited spin state but with the latter (Excited B) being more likely.* Since both cases have f_ϕ / f_ψ >1, both LAM and SAM states are possible and in the next section we give arguments why a SAM state is preferred.

V. THE SPIN STATE OF 2P/ENCKE

The choice of a low excitation SAM state. The full definition of a spin state requires specification of eight parameters at a particular time. A convenient set are the two frequencies associated with the precession of the long axis and rotation about the long axis f_ϕ and f_ψ, the direction of the total angular momentum vector, the two ratios of the moments of inertia about the principal axes, and the initial values of ϕ and ψ. The basic frequencies have been determined in the previous section, and here we will argue that the direction of the angular momentum direction is already known through a generalization of the work of Sekanina (1988*a, b*) and Festou and Barale (2000). Crude information on the ratio of the moments of inertia (or, alternatively, the ratio of the lengths of the principal axes when homogeneity of the interior mass distribution the nucleus is invoked and the excited mode (SAM or LAM) is specified) is available from the amplitude of the 10.7 μm light curve. However, an assessment of meaningful initial values for ϕ and ψ is not possible with the current data set.

Sekanina (1987,1988*a, b,* 1991*a*) and Festou and Barale (2000) have derived similar directions for the spin axis of an assumed spherical nucleus for 2P/Encke from observations of the fan-like morphology of the visible coma (thought to be primarily light fluoresced by C_2 and/or CN molecules) and, in the latter case, from the distribution of light in the OH coma. During its evolution, the cone angle of the visible fan varies between 50 and 120 deg. They find the spin axis, which for a spherical rotator is coincident with the rotational angular momentum vector **M**, is in the direction (α,δ) = (205.6, 1.5 deg.) and (198.6, -0.3 deg.) respectively (we have precessed their results to the J2000 epoch). Since the Festou and Barale result is in essence a fine tuning of Sekanina's pole determination we accept their result as the best current estimate. In both of these studies an explanation of either the distribution of light in the visible coma (Festou and Barale, 2000) or the evolution of the geometric orientation of the coma fan (Sekanina, 1988*a, b*) is consistent with the presence of a predominant active area on the nucleus at a high cometocentric latitude (+55 deg.) in the northern hemisphere (for a sphere there is no distinction between cometocentric and cometographic latitude). In addition Sekanina's work implies an additional active area in the southern hemisphere, again at high cometocentric latitude (-75 deg.). To apply Sekanina's and Festou and Barale's results to our concept of the nucleus as an elongate object we must first generalize their analysis to the case of an asymmetric elongate rotator.

For an elongate nucleus in principal axis rotation the issue is trivial since the local geometry of a small area on a spherical nucleus can always be duplicated (*cf.* Fig. 5b for visualization of the geometry) on the elongate surface. Mid-latitude regions on a spherical nucleus could, for example, have similar emission geometry to regions near the ends of the nucleus. High-latitude areas on a spherical nucleus could map into regions far from the rotational angular momentum vector on a particularly elongated object and still be able to satisfy the observations. The key element is that the cometo*graphic* latitude be the same. For excited elongated rotators the situation becomes more complex. In the case of low excitation SAM's where the amplitudes of the oscillations about the long axis and its nodding motion are small, the calculation of emission cone angles would have to be modified but locations (Fig. 5c) with essentially equivalent local geometry could still be found. In this case it makes more sense to relate the symmetry axis of the coma fans to **M** (the rotational angular momentum vector) rather that the spin axis, since the instantaneous spin axis is freely precessing around **M**. In the case of high excitation SAM's, where the oscillatory amplitudes are large, and in LAM's, where there are complete rotations around the long axis, the fan-like characteristics would become hopelessly blurred and regions with effectively equivalent insolation properties will not, except for extraordinarily special cases (see below), be available. For example, the LAM state for the frequencies in Models A and B have the angle between the long axis and **M**, *i.e.* $\theta \geq 77$ deg, which cannot be geometrically consistent with the formation of a fanlike coma. The existence of a coma fan with the properties documented by Sekanina *represents the first argument why 2P/Encke (and other periodic comets that show a similar coma fan structure) is likely to either be in a relatively low excitation SAM state or a state of principal axis rotation.* A second argument why the excited states that we have found for 2P/Encke are most likely be SAM's concerns a related issue: the observed asymmetry in water production around perihelion when the heliocentric distance is between 0.7 and 1 AU (A'Hearn *et al.* 1985; Mäkinen *et al.* 2001). This has been considered in detail by Sekanina (1991*a*) who shows convincingly that such an asymmetry naturally arises as the result of the positions of the active regions deduced in his study of the coma fan morphology and the geometrical relationship between the rotation pole geometry and the pole of the comet's orbit. Sekanina's arguments will continue to apply for an elongate nucleus provided that the nucleus is in principal axis rotation or in a low excitation SAM state. For LAM states the asymmetry might still be satisfied but only in the special case of a highly excited LAM (Fig. 5d) where the angle between the instantaneous spin axis and M is about 15 - 35 deg (the range of co-latitudes of Sekanina's active areas) and providing Sekanina's active areas are at the two ends of the elongated nucleus. However, we discount this possibility since with such a placement of the active areas the expected reaction torques would be unlikely to stimulate such a highly excited state in the first place. *We conclude from this discussion of coma structure that the best current estimate of the direction of **M** in 2P/Encke is near α_M, δ_M (J2000) = 198.6, -0.3 deg. and that only the low excitation SAM versions of Models A and B are likely possibilities.*

Limits on the shape of the nucleus. Given a SAM spin state the values of f_ϕ and f_ψ in Models A and B allow us to place a limit on the possible dimensions of the intermediate axis (we prefer to work in the equivalent dimensions of a homogeneous nucleus rather than moments of inertia. This is justified in the work of Belton *et al.,* 1991, who showed that the physical dimensions observed for the nucleus of 1P/Halley taken together with their model for the comet's spin state are consistent with a homogeneous interior mass distribution). Following Samarasinha and A'Hearn (1991) we have in the case of a SAM (their equation A79):

$$P\psi/P\phi \geq \sqrt{(a^2 + c^2)(b^2 + c^2)/(a^2 - c^2)(b^2 - c^2)} \tag{5}$$

This relationship allows us to put a lower limit on the value of b/c since we have already found from the amplitude of the 10.7μm light curve that a/b = 2.04. From the frequency analysis, models A and B are characterized by $P\psi/P\phi$ = 2.99 and 4.31 which gives b/c > 1.18 and 1.09 respectively implying a substantial deviation from axial symmetry in 2P/Encke.

VI. DISCUSSION AND CONCLUSIONS

Available R-band and 10.7 μm observations of 2P/Encke have been analyzed for the presence of coma and frequency content. Brightness variations in the R-band photometry are found to be dominated by variations in coma activity with, at most, small contribution from light scattered directly off the nucleus. Changes in the harmonic content of the light curves are proposed to be due to changes in the relative strengths of active regions that are the ultimate source of the coma. The 10.7 μm observations are least affected by coma (Fernández *et al.* 2000) and play a crucial role in determining the possible spin states and the elongation of the nucleus (a/b = 2.04). An array of seven related periodicities has been found in the data (Table 2), and we have determined the most likely spectral modes that occur in the light curves (Table 4). Principal axis rotation, while possible,is considered unlikely. The total angular momentum vector **M** points parallel to the direction α_M, δ_M = 198.6, -0.3 deg (J2000 equatorial coordinates) based on a generalization of the work of Sekanina (1988a, b) and Festou and Barale (2000). This direction is probably uncertain by as much as 10 deg in each coordinate. The number of discrete active areas remains the same as deduced by Sekanina, *i.e.,* two, but the generalization to an elongate shape places looser constraints on the position of the active areas relative to the angular momentum vector, *i.e.,* they remain at high cometographic latitude but not necessarily at high cometocentric latitude. The sense of the rotational angular momentum vector is not determined. The persistent presence of a fan coma is used as evidence that the nucleus is most likely in a low excitation SAM state. Using a least-squares approach to fitting spin models to the periodicities we find that the nucleus is likely to be found in one of two low-excitation SAM states: Model A with f_ϕ = 1.614 d^{-1} ($P\phi$ = 14.9 h) and f_ψ = 0.539 d^{-1} ($P\psi$ = 44.5 h) and Model B with f_ϕ = 2.162 d^{-1} ($P\phi$ = 11.1 h) and f_ψ = 0.502 d^{-1} ($P\psi$ = 47.8 h) with the latter more likely. Each pair of frequencies together with a/b=2.04 places a constraint on the length of the intermediate axis. We find b/c > 1.18 and 1.09 for model A and B respectively which implies considerable asymmetry in the nucleus shape. Acceptable values of b/c cannot be much larger than these limits otherwise the oscillation amplitudes would be too large and blur out any fan morphology. For example, in Model A with b/c=1.2 the amplitude of the oscillation about the long axis is an acceptable 28 deg and the amplitude of the nodding of the long axis would be 4.8 deg. Increasing b/c to 1.6 would increase these amplitudes to the probably unacceptable levels of 72 and 12 deg respectively. Knowledge of the shape and spin state could be greatly improved through a comprehensive set of well-sampled thermal light curves of the comet nucleus analogous to those made at 10.7 μm by Fernández *et al.* (2000) but expanded to cover a wide range of orbital configurations.

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References

A'Hearn, M.F., Birch, P.V., Feldman, P.D., and Millis, R.L. 1985, Comet Encke – Gas Production and Light curve. *Icarus* **64**, 1 – 10.

Barker, E.S., Cochrane, A.L., and Rybski, P.M. 1981. Observations of faint comets at McDonald Observatory: 1978-1980. In *Modern Observational Techniques for Comets*. Ed. J.C. Brandt *et al.,* JPL Publ. 81-68.

Belton, M.J.S. 1990. Rationalization of Halley's Periods. *Icarus* **86**, 30-51

Belton, M.J.S. 1991. Characterization of the Rotation of Cometary Nuclei. In *Comets in the Post-Halley Era*, ed. R.L. Newburn Jr., M. Neugebauer, and J. Rahe (Dordrecht: Kluwer), **2**, 691-721.

Belton, M.J.S., Wehinger, P., Wyckoff, and Spinrad, H. 1986. A Precise Period for P/Halley. ESA SP-250, *Exploration of Halley's Comet* **2**, 599-603.

Belton, M.J.S., Mueller, B.E.A., Julian, W.H., and Anderson, A.J. 1991. The Spin State and Homogeneity of Comet Halley's Nucleus. *Icarus* **93**, 183-193.

Delsemme, A. 1982. Chemical composition of cometary nuclei. In *Comets* (L. L. Wilkening, ed.), pp. 85-130. Univ. of Arizona, Tucson.

Fernández, Y.R., Lisse, C.M, Käufl, H.U., Pescke, S.B., Weaver, H.A., A'Hearn, M.F., Lamy, P.P., Livengood, T.A., and Kostiuk, T. 2000. Physical properties of the Nucleus of Comet 2P/Encke. *Icarus* **147**, 145-160.

Fernández, Y.R., Lowry, S.C., Weissman, P.R., Mueller, B.E.A., Belton, M.J.S., and Meech, K.J. 2004. New Near-Aphelion Light Curves of Comet 2P/Encke, *Icarus,* submitted.

Festou, M.C., and Barale, O. 2000. The Asymmetric Coma of Comets. I. Asymmetric Outgassing from the Nucleus of Comet 2P/Encke. *Astron. J.* **119**, 3119-3132.

Foster, G. 1995. The Cleanest Fourier Spectrum. *Astron. J.* **109**, 1889-1902.

Jewitt, D. 1997. Cometary Rotation: An Overview. *Earth, Moon and Planets* **79**, 35-53.

Jewitt, D., and Danielson, G.E. 1984. Charge Coupled Device Photometry of Comet P/Halley. *Icarus* **60**, 435-444.

Jewitt, D. and Meech, K.J. 1987. CCD Photometry of Comet P/Encke, *Astron. J.* **93**, 1542-1548.

Kaasalainen, M. 2001. Interpretation of Light Curves of Precessing Asteroids. *Astron. Astrophys*. **376**, 302-309.

Keller, H.U., Curdt, W. Kramm, J.-R., and Thomas, N. Images of the Nucleus of Comet Halley. *ESA SP-1127*, Vol. 1.

Lamy, P.L., Toth, I., Fernandez, Y.R., Weaver, H.A. 2004. The Sizes, Shapes, Albedos, and Colors of Cometary Nuclei. In *Comets II.* Ed. M.C. Festou *et al.* In press.

Landolt, A. U. 1992. UBVRI photometric standards in the magnitude range 11.5-16.0 around the celestial equator, *Astron. J.* **104**, 340-371.

Leibowitz, E.M., and Brosch, N. 1986. Photoelectric Discovery of a 52 hr Periodicity in the Nuclear Activity of P/Halley. *Icarus* **68**, 418-429.

Licandro, J., Tancredi, G., Lindgren, M., Rickman, H., and Hutton, R.G. 2000.CCD Photometry of Cometary Nuclei, I: Observations from 1990-1995. *Icarus* **147**, 161-179.

Lowry, S.C., Weissman, P.R., Sykes, M.V., and Reach, W.T. 2003. Observations of Periodic Comet 2P/Encke: Physical properties of its Nucleus and the first visual-wavelength Detection of its Dust Trail. In *Lunar and Planetary Science XXXIV* (Houston: Lunar Planet. Inst.), No. 2056.

Luu, J., and Jewitt, D. 1990. The Nucleus of Comet P/Encke. *Icarus* **86**, 69-81.

Mäkinen, J.T.T., Silén, J., Schmidt, W., Kyrölä, E., Summanen, T., Bertaux, J.-L., Quémerais, E., and Lallement, R. 2001. Water Production of Comets 2P/Encke and 81P/Wild 2 Derived from SWAN Observations during the 1997 Apparition. *Icarus* **152**, 268 – 274.

McFadden, L.A., A'Hearn, M.F., Feldman, P.D., Roettger, E.E., Edsall, D.M., and Butterworth, P.D. 1987. *Astron. Astrophys*. **187**, 333-338.

Meech, K.J., Belton, M.J.S., Mueller, B.E.A., Dicksion, M.W., and Heide, R.Li 1993. Nucleus Properties of P/Schwassmann-Wachmann 1. *Astron. J.* **106**, 1222-1236.

Meech, K.J., Fernandez, Y., and Pittichova, J. 2001. Aphelion Activity of 2P/Encke. *Bull. Amer. Astron. Soc.* **33**, 1075.

Meech, K.J., Fernandez, Y.R., Pittichova, J., Bauer, J.M., Hsieh, H., Belton, M.J.S., A'Hearn, M.F., Hainaut, O.R., Boehnhardt, H., and Tozzi, G.P. 2002. Deep impact Nucleus Characterization – A Status Report. *Bull. Amer. Astron. Soc.* **34**, 870.

Meech, K. J. and Svoren, J. 2004. Using Cometary activity to trace the physical and chemical evolution of cometary nuclei, in *Comets II*. Ed. M.C. Festou *et al*. In press.

Millis, R.L., and Schleicher, D.G. 1986. Rotational period of Comet Halley. *Nature* **324**, 646-649.

Mueller, B.E.A., and Ferrin, I. 1996. Change in the Rotational period of Comet P/Tempel 2 between the 1988 and 1994 Apparitions. *Icarus* **123**, 463 – 477.

Mueller, B.E.A., Samarasinha, N.H., and Belton, M.J.S. 2002. The Diagnosis of Complex Rotation in the Light curve of 4179 Toutatis and potential Applications to other Asteroids and Bare Cometary Nuclei. *Icarus* **158**, 305-311.

Neishtadt, A.I., Scheeres, D.J., Siderenko, V.V., and Vasiliev, A.A. 2002. Evolution of Comet Nucleus Rotation. *Icarus* **157**, 205-218.

Reach, W.T., Sykes, M.V., Lien, D., and Davies, J.K. 2000. The Formation of Encke Meteoroids and Dust Trail. *Icarus* **148**, 80-94.

Roemer, E. 1962. Activity in Comets at Large Heliocentric Distance. *Pub. Astron Soc. Pacific* **74**, 351-365.

Samarasinha, N.H. 1997. Preferred Orientations for the Rotational Angular Momentum Vectors of Comets. *Bull. Am. Astron. Soc*. **29**, 743-743.

Samarasinha, N.H. 2003. Cometary Spin States, their Evolution, and the Implications. In *Cometary Nuclei in Space and Time: Proceedings of IAU Colloquium 168* (M.A. A'Hearn, ed.) ASP Conference Series, in press.

Samarasinha, N.H., and A'Hearn, M.F. 1991. Observational and Dynamical Constraints on the Rotation of Comet P/Halley. *Icarus* **93**, 194-225.

Samarasinha, N.H., and Belton, M.J.S. 1995. Long-Term Evolution of Rotational States and Nongravitational Effects for Halley-like Cometary Nuclei," 1995. *Icarus,* **116**, 340 - 358.

Samarasinha, N.H., Mueller, B.E.A., Belton, M.J.S., and Jorda, L. 2004. Rotation of Cometary Nuclei. In *Comets II*. Ed. M.C. Festou *et al*. In press.

Soderblom, L.A., Becker,T.L., Bennett,G., Boice, D.C., Britt, D.T., Brown, R.H., Burratti, B.J., Isbell, C., Giese, B., Hare, T., Hicks, M.D., Howington-Kraus, E., Kirk, R.L., Lee, M., Nelson, R.M., Oberst, J., Owen, T.C., Rayman, M.D., Sandel, B.R., Stern, S.A., Thomas, N., Yelle, R.V. 2002. Observations of Comet 19P/Borrelly by the Miniature Integrated Camera Aboard deep Space 1. *Science* **296**, 1087-1091.

Szegö, K., Sagdeev, R.Z., Whipple, F.L., Abergel, A., Bertaux, J.-L., Merényi, E.,Szalai, S., and Várhalmi, L. 1994. Images of the Nucleus of Comet Halley. *ESA SP-1127*, Vol. 2.

Sekanina, Z. 1987. Anisotropic Emission from Comets I: Concept and Modeling. In *Diversity and Similarity of Comets.* ESA-SP 278, 315 – 322.

Sekanina, Z. 1988*a*. Outgassing Asymmetry of Periodic Comet Encke. I. Apparitions 1924-1984. *Astron. J.* **95,** 911-924

Sekanina, Z. 1988*b*. Outgassing Asymmetry of Periodic Comet Encke. II. Apparitions 1868-1918 and a Study of the Nucleus Evolution. *Astron. J.* **95**, 911-924

Sekanina, Z. 1991a. Encke, the Comet. *J. Roy. Astron. Soc. Canada* **85**, 324-376.

Sekanina, Z. 1991b. Comet Activity, Discrete Outgassing Areas, and Dust Jet Formation. In *Comets in the Post-Halley Era*, ed. R.L. Newburn Jr., M. Neugebauer, and J. Rahe (Dordrecht: Kluwer), **2**, 691-721.

Soderblom, L.A., Boice, D.C., Britt, D.T., Brown, R.H., Buratti, B.J., Kirk, R.L., Lee, M., Nelson, R.M., Oberst, J., Sandel, B.R., Stern, S.A., Thomas, N., and Yelle, R.V. 2004, Imaging Borrelly, *Icarus*, **167**, 4-15.

Wyckoff, S., Wagner, R.M., Wehinger, P.A., Schleicher, D.G., and Festou, M.C. 1985. Onset of Sublimation in Comet P/Halley. Nature 316, 241-242.

West, R.M., and Jorgensen, H.E. 1984. Post-perihelion Observations of Comet Halley at r = 8.5AU. *Astron. Astrophys.* **218**, 307-316.

West, R.M., and Pedersen, H. 1989. Variability of P/Halley. *Astron. Astrophys* **138**, L9-L10.

Table 1: Observational and Calculated Parameters for the 2P/Encke Data**

**Data Set (R-band unless stated otherwise)	Date	Range of Heliocentric Distance (AU)	Range of Phase Angle (deg.)	*Mean $H_R(1,1,0)$ (mag)	Observed Light Curve Range (mag)	++Ratio of Nucleus to Total Brightness	+++ Polar Aspect Angle (deg)	+Diluted Nucleus Light Curve Range (mag)
Jewitt and Meech (1987)	Sept 1985	4.06	6.6 - 6.9	13.81	>0.88	0.14 – 0.50 (0.28)	46.7	0.15
Jewitt and Meech (1987)	Oct/Nov 1986	3.13 – 3.15	14.4 – 15.4	14.42	>0.35	0.27 – 0.88 (0.49)	35.4	0.19
Luu and Jewitt (1990)	Aug/Sep 1988	3.82 – 3.83	3.2 – 4.5	14.32***	0.62	0.24 – 0.80 (0.45)	49.7	0.25
Fernandez et al. (2004)	July 2001	3.433	11.1	13.82	>0.32	0.14 – 0.51 (0.28)	40.2	0.13
Fernandez et al. (2004)	Aug 2001	3.53 – 3.548	3.8 – 2.8	14.33	0.47	0.24 – 0.81 (0.45)	46.6	0.24
Fernandez et al. (2004)	Sept 2001	3.701 –3.715	9.6 – 10.5	14.77	0.85	0.42 – 1.0 (0.67)	57.4	0.42
Fernandez et al. (2004)	Oct 2001	3.754 – 3.76	13.6 – 12.9	15.05	>0.5	0.38 – 1.0 (0.87)	59.3	0.56
Fernandez et al. (2004)	Sept 2002	3.97 – 3.961	0.9 – 2.3	13.89	0.41	0.15 – 0.54 (0.30)	33.2	0.11
Fernandez et al. (2000) 10.7 μm	July 1997	1.16 - 1.257	50.3 – 40.3	-	0.7	-	66.0	-

*Assumes a phase coefficient of 0.06 mag/deg (Fernández *et al.* 2000); ** The Jewitt and Meech data is treated as two separate data sets, while the very large data set in Fernández et al. (2004) is divided into five segments. *** To be consistent, Luu and Jewitt's estimate of the mean absolute magnitude has been adjusted to a phase law of 0.06 mag/deg. + With the coma background level assumed constant. ++ Values in parenthesis are probable values of nucleus light to total light. +++ Assumes a pole direction of α, δ *(J2000) = 198.6, -0.3 deg.* - see text (section V).

Table 2: Periodicities in the Light Curves of 2P/Encke

Authors	**Date**	**Frequencies (d^{-1})**						
Luu & Jewitt (1990)	9/1988	-	-	-	***3.19***		-	8.5:
Jewitt & Meech (1987)	9/1985	0.46	-	***2.16***	3.18	4.35	-	8.75
Jewitt & Meech (1987)	10/1986	-	-	-	***3.14***	-	-	8.70
Fernández *et al* (2000)	7/1997	0.60	-	-	3.27	***4.27***	5.23	8.5:
Fernández *et al* (2004)	8/2001	-	-	***2.17***	-	4.36	-	-
Fernández *et al* (2004) (Computed by Mueller)	9/2001	-	1.17	-	***3.24***	-	5.34	-
Fernández *et al* (2004)	9/2002	-	-	***2.15***	-	4.34	-	-
Averaged frequencies:		**0.53 ±0.11**	**1.17 ±0.16**	**2.16 ±0.09**	**3.20 ±0.07**	**4.33 ±0.08**	**5.29 ±0.11**	**8.65 ±0.09**
Model A assignments (excited state)		f_ψ	$(f_\phi - f_\psi)$	$2(f_\phi - f_\psi)$	$2f_\phi$	$2(f_\phi + f_\psi)$	$3f_\phi + f_\psi$	$4(f_\phi + f_\psi)$
Model A frequencies (f_ϕ= 1.614; f_ψ= 0.539)		0.54	1.13	2.15	3.22	4.30	5.38	8.61
Model B assignments (excited state)		f_ψ	f_ϕ-$2f_\psi$	f_ϕ	$f_\phi + 2f_\psi$	$2f_\phi$	$2(f_\phi + f_\psi)$	$4f_\phi$
Model B frequencies (f_ϕ 2.162, f_ψ = 0.502)		0.50	1.16	2.16	3.17	4.32	5.35	8.65
Model assignments (fully relaxed state)		f_ϕ	$2f_\phi$	$4f_\phi$	$6f_\phi$	$8f_\phi$	$10f_\phi$	$16f_\phi$
Model frequencies (f_ϕ = 0.5381)		0.54	1.08	2.15	3.25	4.3	5.38	8.61

Italics denote the strongest peak. The uncertainty of the individual entries is estimated at ±0.16 d^{-1} (except for those marked with a colon for which we take ±0.2 d^{-1}). In line 9 the quoted uncertainty is the uncertainty of the mean. Values followed by a colon are particularly uncertain.

Table 3: Periodicities in the Light Curves of 1P/Halley

Authors	Frequencies (d^{-1})						
Millis & Schleicher (1986)	0.13	***0.27***	0.41	-	-	-	-
McFadden *et al.* (1987)	-	0.29	***0.41***	-	-	-	-
PrePerihelion*	***0.13***	-	0.43	0.54	-	-	-
Leibowitz and Brosch (1986)	-	***0.22*:**	-	0.51:	-	-	-
West & Jorgensen (1989)	-	-	-	-	1.11	2.27	***6.25***
Averaged Frequencies	**0.13**	**0.27**	**0.42**	**0.53**	**1.11**	**2.27**	**6.25**
Model Assignments (excited state)	$(f_\phi - f_\psi)$	f_ϕ	$(f_\phi + f_\psi)$	$2f_\phi$	$4f_\phi$	$8f_\phi$	$22f_\phi$
Model Frequencies (f_ϕ = 0.28; f_ψ = 0.15)**	0.14	0.28	0.43	0.57	1.14	2.27	6.24
Model Assignments (fully relaxed spin)	f_ϕ	$2f_\phi$	$3f_\phi$	$4f_\phi$	$8f_\phi$	$16f_\phi$	$44f_\phi$
Model Frequencies (f_ϕ = .142)	0.14	0.28	0.43	0.57	1.14	2.27	6.24

Italics denote the strongest frequency. : Denotes considerable uncertainty; * 70 pre-perihelion observations that include the work of Jewitt & Danielson (1984), West and Pedersen (1984) and others (*cf.* Belton *et al.* 1986). ** Note that the solution given here for the basic periodicities is solely based on Earth-based observations and is slightly different from the solution based on the spacecraft observations alone given in Belton et al. (1991), *i.e.*, f_ϕ = 0.27; f_ψ = 0.14 d^{-1}.

Table 4. Least Squares Goodness of Fit of Spin Models for 2P/Encke

Model	Spectral Frequencies and Modes							G ; H (x 10^4)
Averaged observed frequencies (d^{-1})	**0.53**	**1.17**	**2.16**	**3.20**	**4.33**	**5.29**	**8.65**	-
Fully relaxed (f_ϕ = 0.5381)	f_ϕ	$2f_\phi$	$4f_\phi$	$6f_\phi$	$8f_\phi$	$10f_\phi$	$16f_\phi$	29 ; 12
Excited A (f_ϕ, f_ψ = 1.614, 0.539)	f_ψ	$f_\phi - f_\psi$	$2(f_\phi - f_\psi)$	$2f_\phi$	$2(f_\phi + f_\psi)$	$3f_\phi + f_\psi$	$4(f_\phi + f_\psi)$	29 ; 12
Excited B (f_ϕ, f_ψ = 2.162, 0.502)	f_ψ	$f_\phi-2f_\psi$	f_ϕ	$f_\phi + 2f_\psi$	$2f_\phi$	$2(f_\phi + f_\psi)$	$4f_\phi$	5 ; 5
Excited C (f_ϕ, f_ψ= 2.152, 0.542)	f_ψ	$2f_\psi$	f_ϕ	$2(f_\phi - f_\psi)$	$2f_\phi$	$2(f_\phi + f_\psi)$	$4f_\phi$	29 ; 10
Excited D (f_ϕ, f_ψ = 1.083, 0.622)	f_ψ	f_ϕ	$2f_\phi$	$2(f_\phi + f_\psi)$	$4f_\phi$	$3(f_\phi + f_\psi)$	$8f_\phi$	130 ; 48

Figure Captions

Figure 1. Cometocentric map of the observational geometry of the R-band and IR observations used in this study. The direction of the observer at each R-band observational epoch is plotted as filled triangles and the plane of the orbit is shown. The cometocentric position of the 10.7 μm observation is shown as an X. Also shown are the direction of the Sun at aphelion (open circle) and the rotational angular momentum direction (filled circle) as determined by studies of coma morphology. These two directions, for which some studies (Samarasinha, 1997, 2003; Neishtadt *et al.,* 2002) predict approximate alignment as the end point of rotational angular momentum evolution in reaction to the flow of material from active areas, are separated by 37.4 degrees. We estimate the uncertainty of the rotational angular momentum direction at roughly ±10 deg in each coordinate. Reach *et al.* (2000) require the spin vector to be parallel to the plane of the orbit at the time of their observations.

Figure 2. Heliocentric dependence of absolute magnitudes 2P/Encke as observed by Meech *et al.* (2001) from the UH 2.2m, KPNO 4.0m and 2.1m telescopes. A phase correction of 0.06 mag/degree has been applied. The data were obtained through the Kron-Cousins R-band filter, and calibrated with standards from Landolt (1992). The ▲ symbols represent pre-perihelion data and the **x** symbols represent the post-perihelion data. The horizontal line at 15.2 mag is the mean nucleus magnitude as estimated by Fernández *et al.* (2000) and the horizontal dashed lines represent the full range of possible mean nucleus magnitudes estimated in this paper. The heliocentric distances corresponding to perihelion and aphelion are also shown. The variability of the comet at aphelion is clearly evident.

Figure 3. (a) WindowCLEAN spectra of various R-band photometric series for 2P/Encke taken over several years when the comet was near aphelion. Also illustrated is a WindowCLEAN spectrum of the 10.7μm observations of Fernández *et al.* (2000). The persistence of the spectral peak near 3.2 d^{-1} should be noted. The primary periodicities derived from these data

are collected in Table 2. (b) These are spectra of the four data sets in (a) after they have been whitened of the periodicity at 3.2 d^{-1}. The underlying frequencies can be more easily seen.

Figure 4. (a) WindowCLEAN spectra of Aug 2001 and Sept 2002 R-band data after Fernandez *et al.* (2004). Note the absence of a strong spectral response near 3.2 d^{-1} in these data, although a weak response at this frequency is evident in the Sept 2002 data. (b) These are spectra of the two data sets in (a) after they have been whitened of the periodicity at 2.16 d^{-1}.

Figure 5. (a) Sekanina's (1991*a*) illustration of his collimation model for the origin of fan comas (reproduced with permission). (b) Generalization to an elongate shape. The active area has the same cometographic latitude but is displaced on the nucleus relative to the spin axis. The base of the cone traced out by the outgassing material is a circle traced out in space by the motion of the active area. (c) Generalization to a low excitation SAM state with an elongate nucleus. Here the long axis of the nucleus nods up and down as it precesses around **M**. Crossed arrows at the end of the cone vector indicate the increasing waviness of the cone surface. However, providing the oscillation about the long axis and the nodding of the long axis is sufficiently small, the model retains its basic character. (d) For a LAM state to emulate the essential characteristics of Sekanina's collimation model a high excitation LAM is required, the nucleus would need to be close to symmetric shape and the active areas would need to be placed near the extreme ends of the nucleus. In this state the long axis nods while it precesses around **M**. In addition, the nucleus now performs full rotations around the long axis.

Figure 1

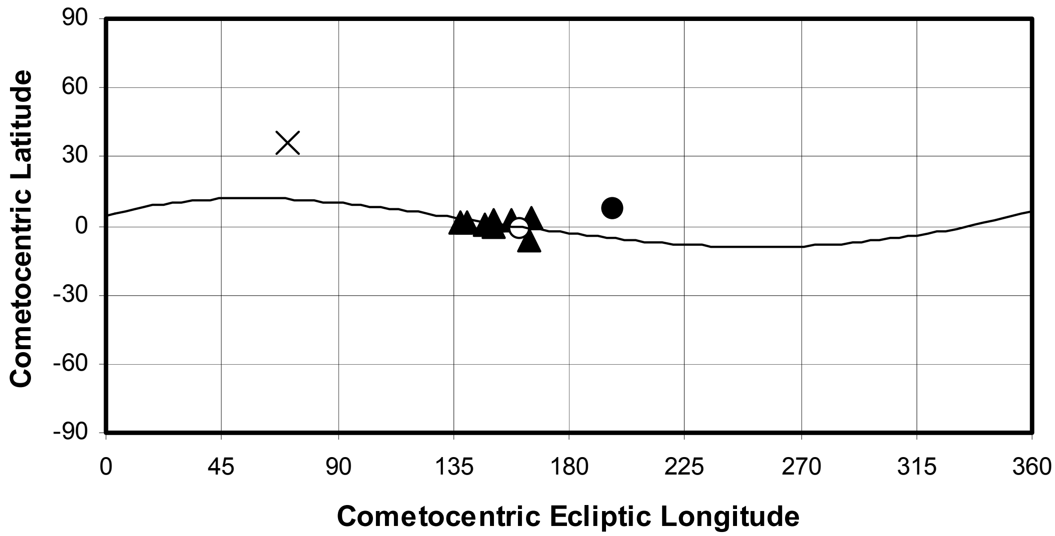
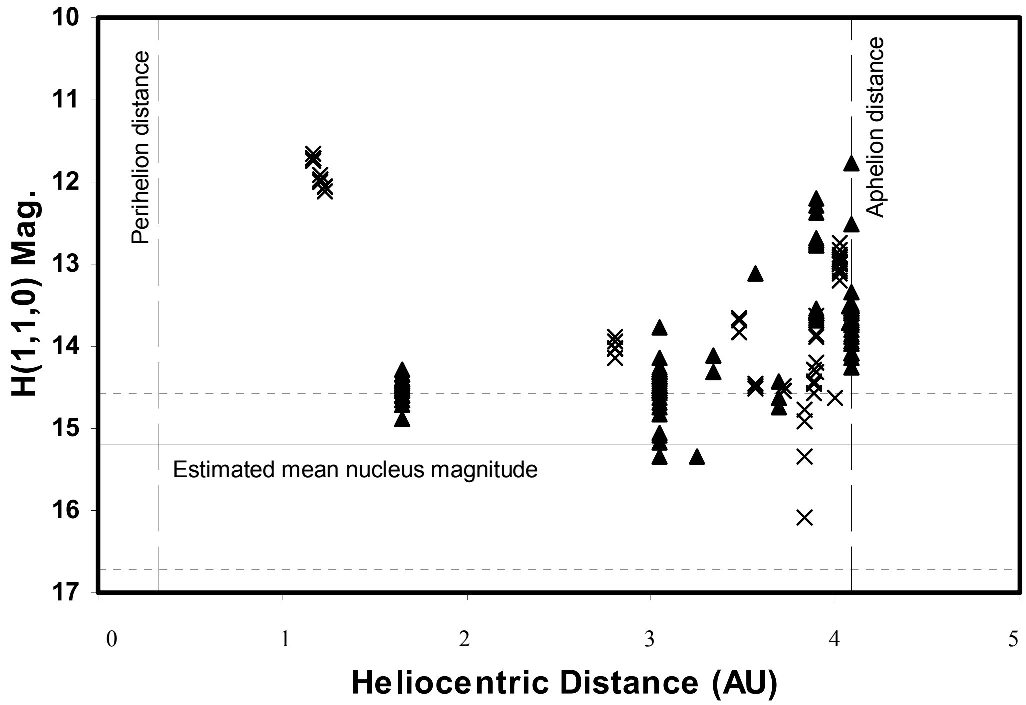

90
60
30
0
-30
-60
-90
0
45
90
135
180
225
270
315
360
Cometocentric Latitude
Cometocentric Ecliptic Longitude

Figure 2



10
11
12
13
14
15
16
17
H(1,1,0) Mag.
Perihelion distance
Aphelion distance
Estimated mean nucleus magnitude
0
1
2
3
4
5
Heliocentric Distance (AU)

Figure 3a

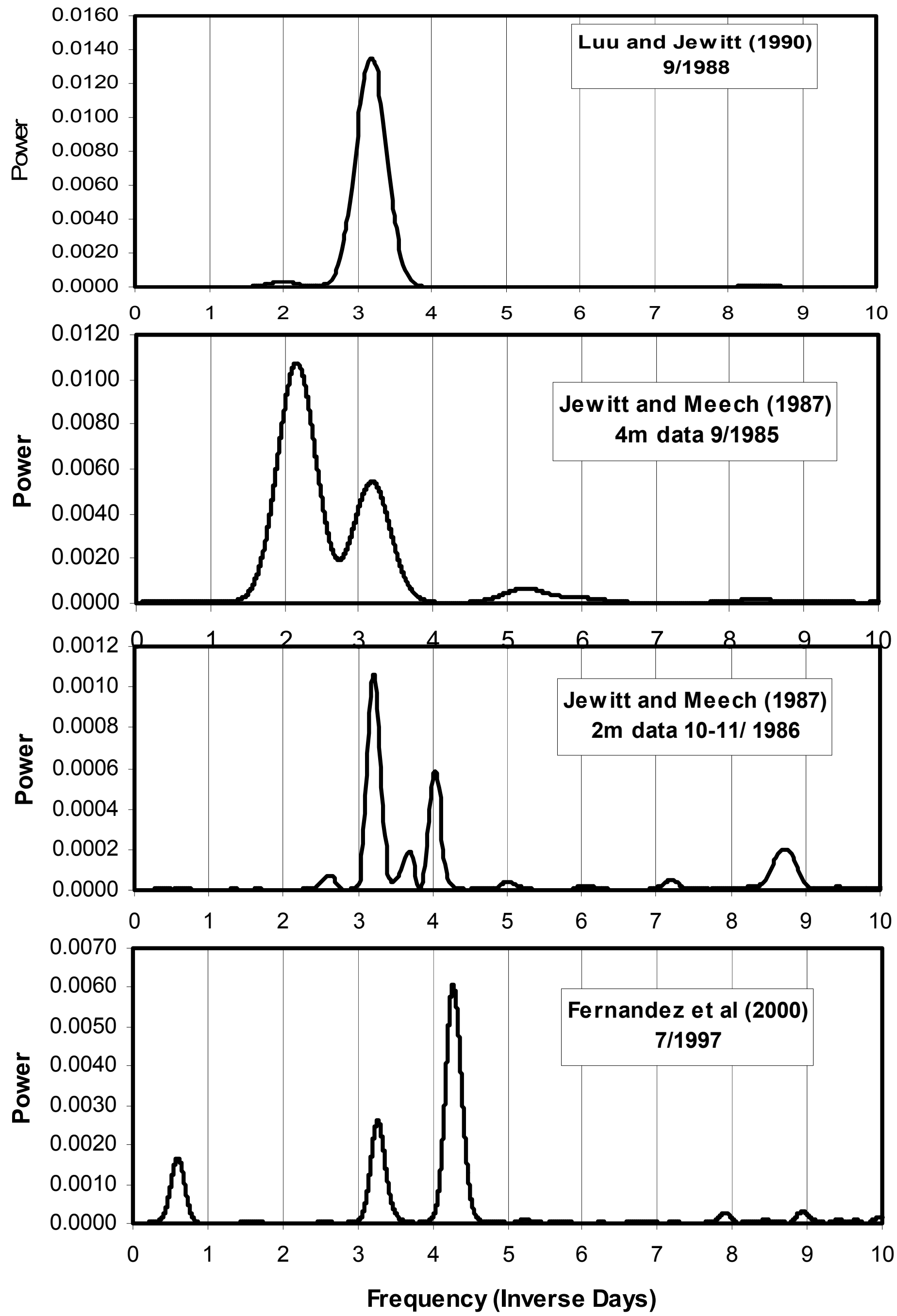

Luu and Jewitt (1990)
9/1988
Jewitt and Meech (1987)
4m data 9/1985
Jewitt and Meech (1987)
2m data 10-11/ 1986
Fernandez et al (2000)
7/1997
Power
Frequency (Inverse Days)

Figure 3b

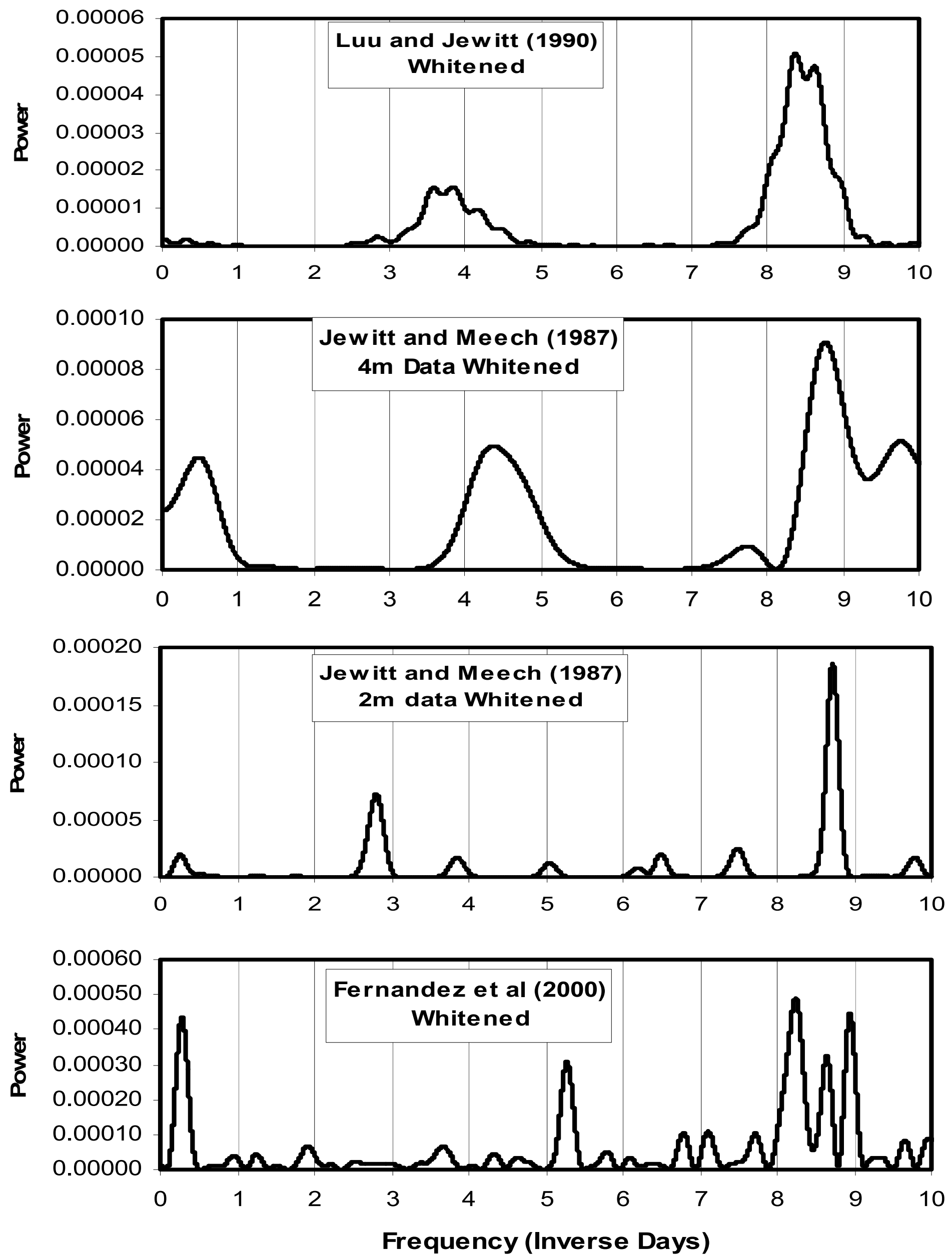

Luu and Jewitt (1990)
Whitened
Jewitt and Meech (1987)
4m Data Whitened
Jewitt and Meech (1987)
2m data Whitened
Fernandez et al (2000)
Whitened
Power
Frequency (Inverse Days)

Figure 4a

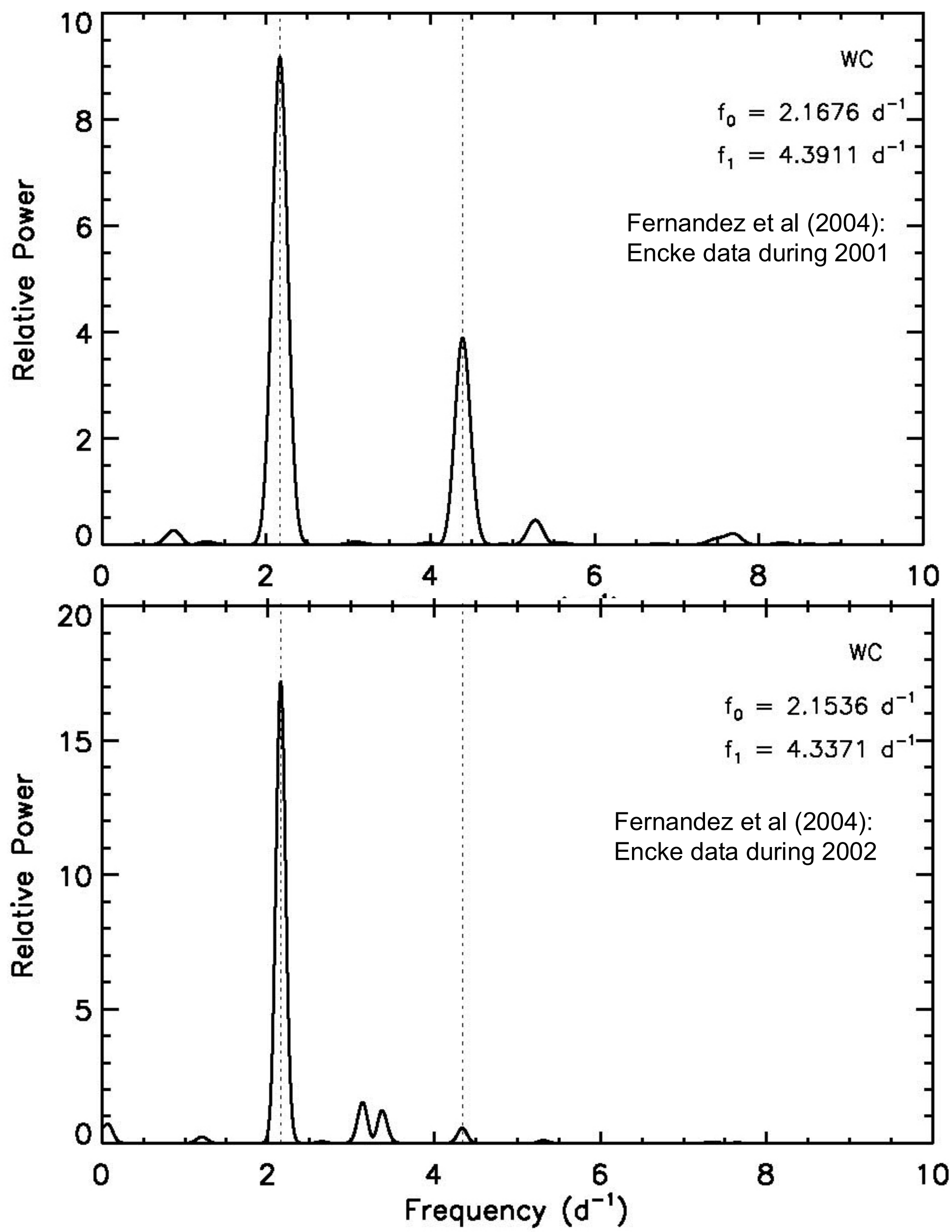

WC
f0 = 2.1676 d-1
f1 = 4.3911 d-1
Fernandez et al (2004):
Encke data during 2001
Relative Power
WC
f0 = 2.1536 d-1
f1 = 4.3371 d-1
Fernandez et al (2004):
Encke data during 2002
Relative Power
Frequency (d-1)

Figure 4b

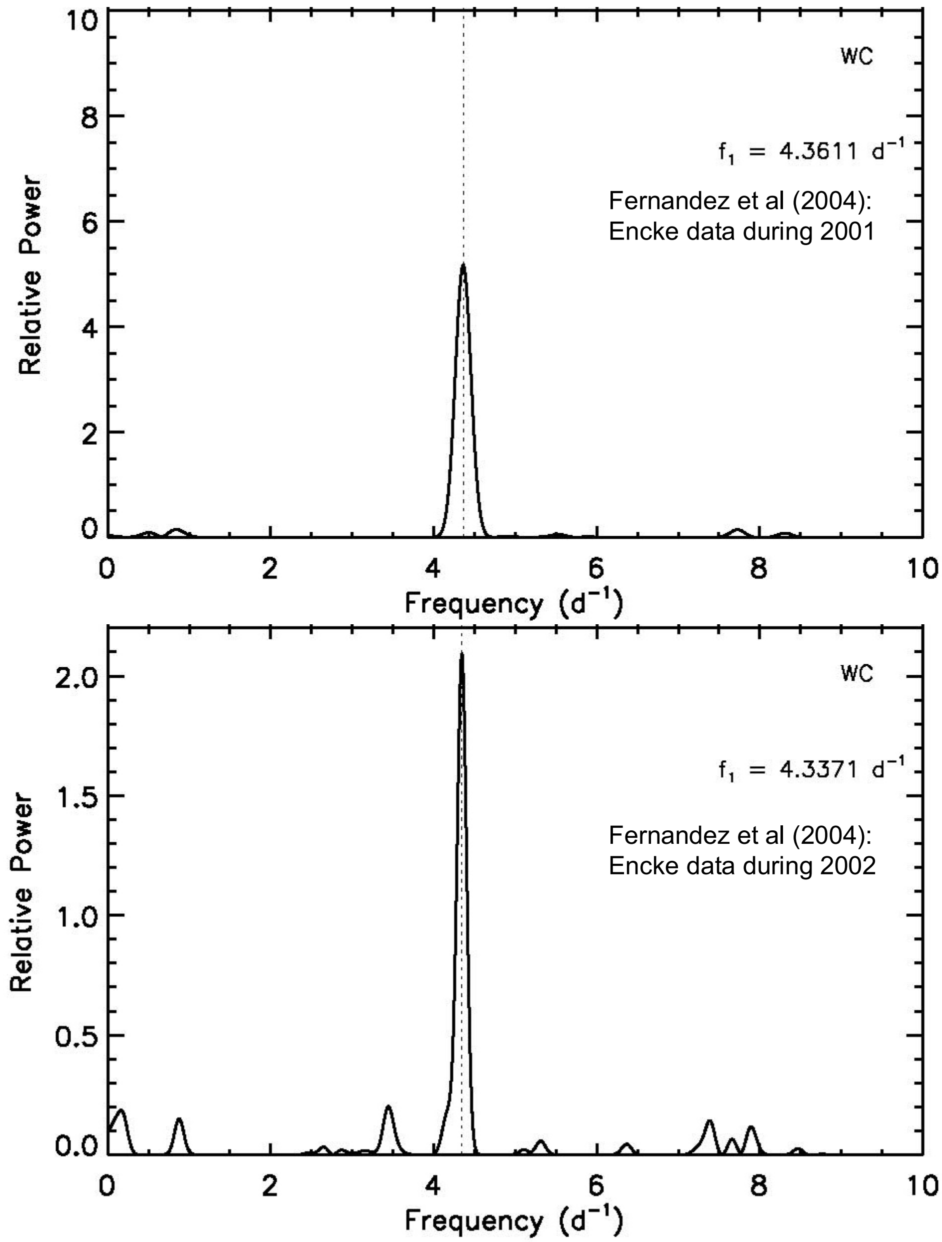
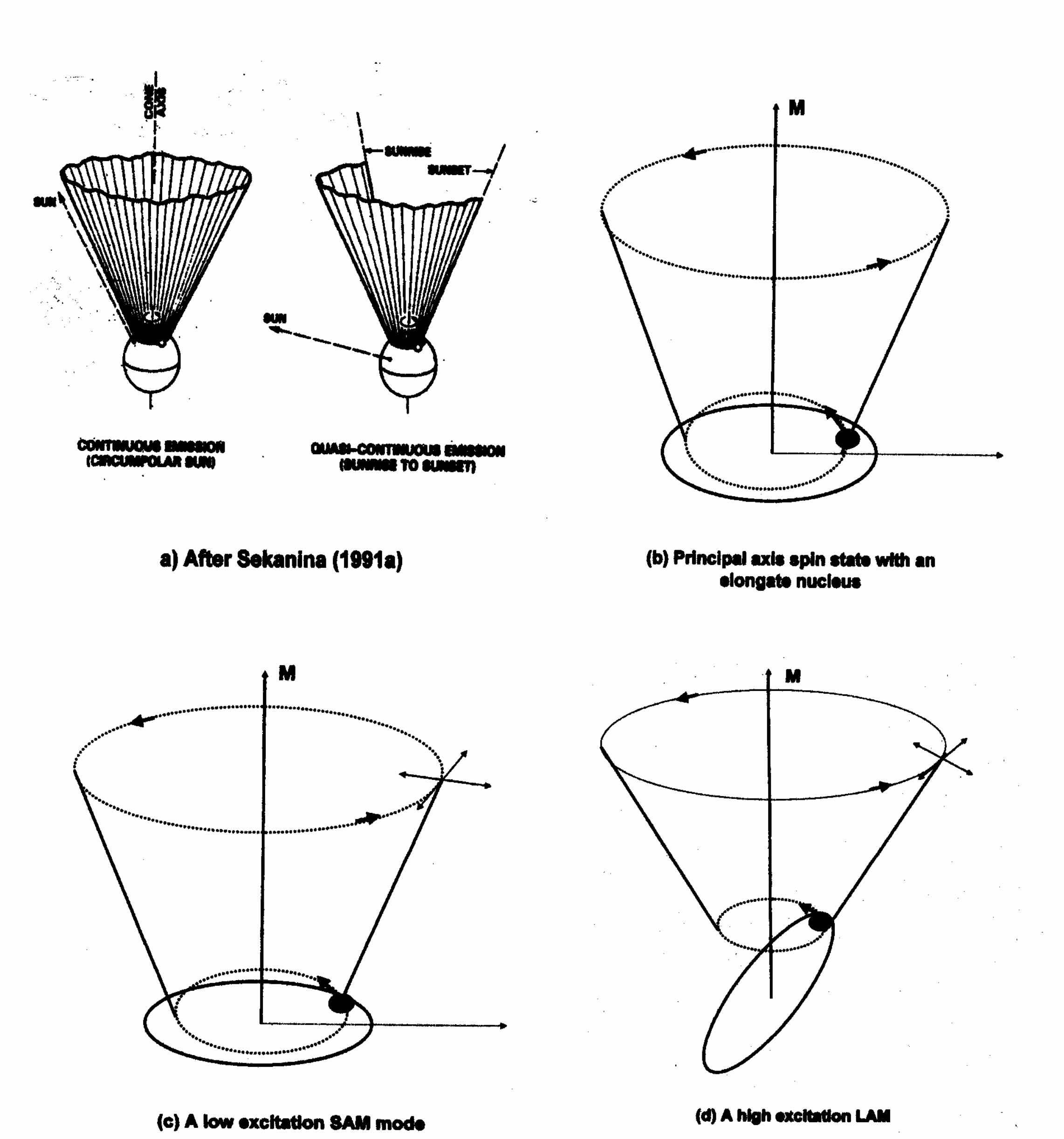

WC
f_1 = 4.3611 d^{-1}
Fernandez et al (2004):
Encke data during 2001
Relative Power
Frequency (d^{-1})
WC
f_1 = 4.3371 d^{-1}
Fernandez et al (2004):
Encke data during 2002
Relative Power
Frequency (d^{-1})

Figure 5



a) After Sekanina (1991a)

(b) Principal axis spin state with an elongate nucleus

(c) A low excitation SAM mode

(d) A high excitation LAM